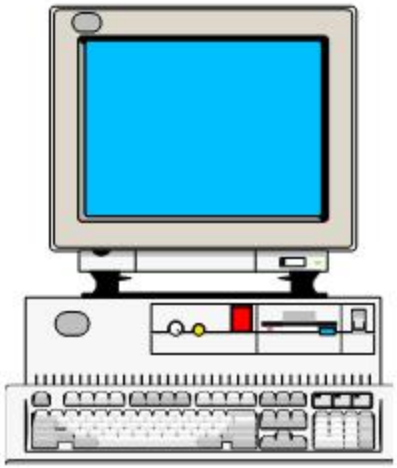
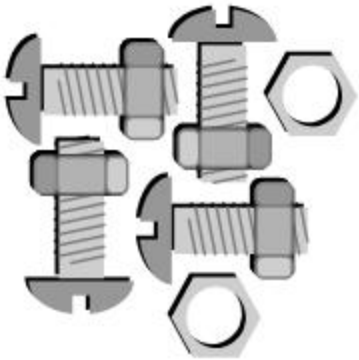
Inventory Control Models

Courtesy , Dr Rafi Javed Qureshi

What is Inventory?

- Stock of materials
- Stored capacity
- Examples





Courtesy , Dr Rafi Javed Qureshi

The Functions of Inventory

- To "decouple" or separate various parts of the production process
- To provide a stock of goods that will provide a "selection" for customers
- To take advantage of quantity discounts
- To hedge against inflation and upward price changes
- Adapting to irregular supply and demand
- Avoiding stock outs and shortages

Courtesy , Dr Rafi Javed Qureshi

Disadvantages of Inventory

- **Higher costs**
  - **Item cost (if purchased)**
  - **Ordering (or setup) cost**

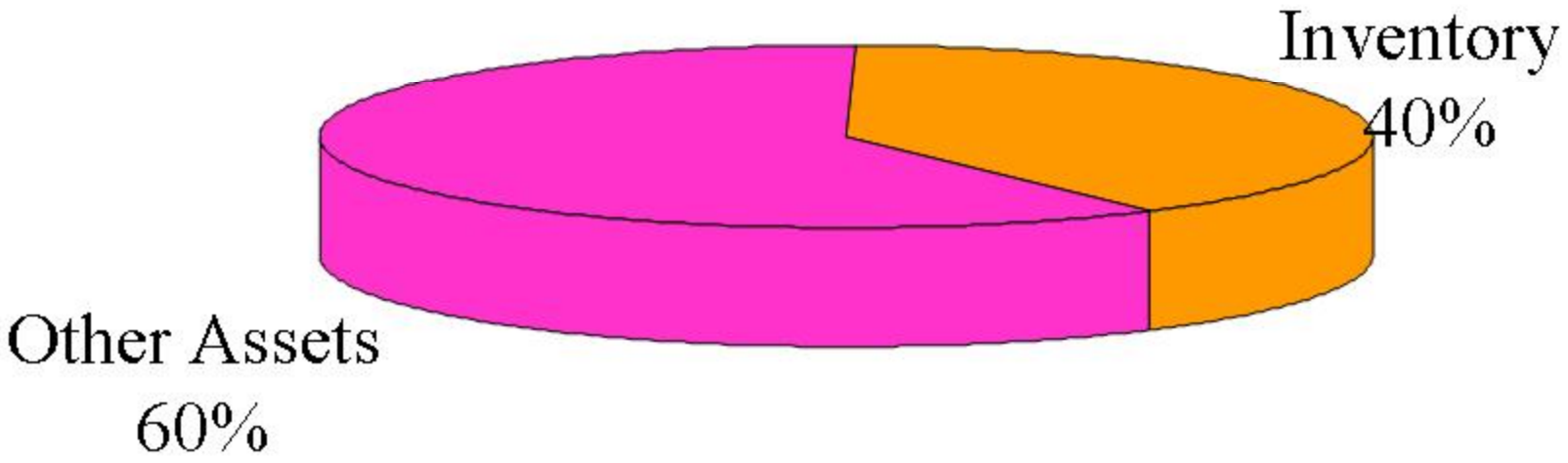
    Costs of forms, clerks' wages etc.
  - **Holding (or carrying) cost**

    Building lease, insurance, taxes etc.
- **Difficult to control**
- **Hides production problems**

Courtesy , Dr Rafi Javed Qureshi

Inventory as an Important Asset

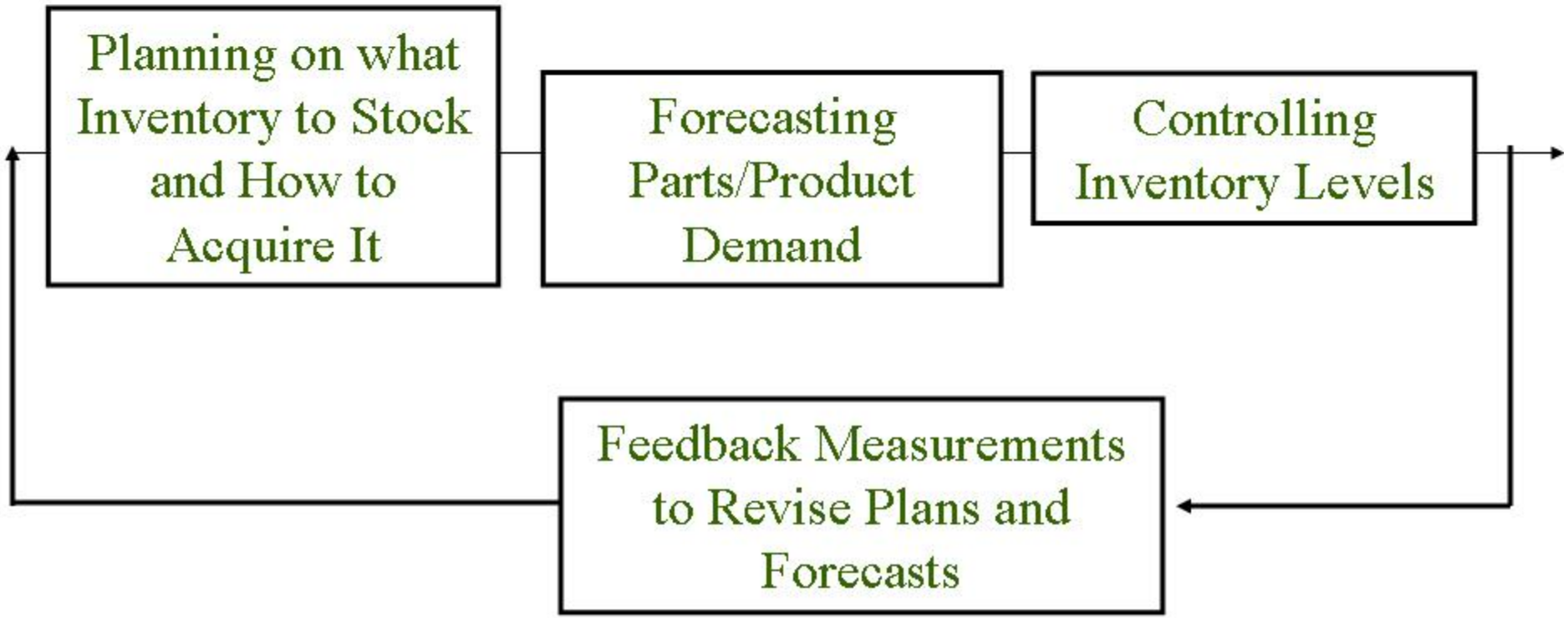
- Inventory can be the most expensive and the most important asset for an organization



Inventory as a percentage of total assets

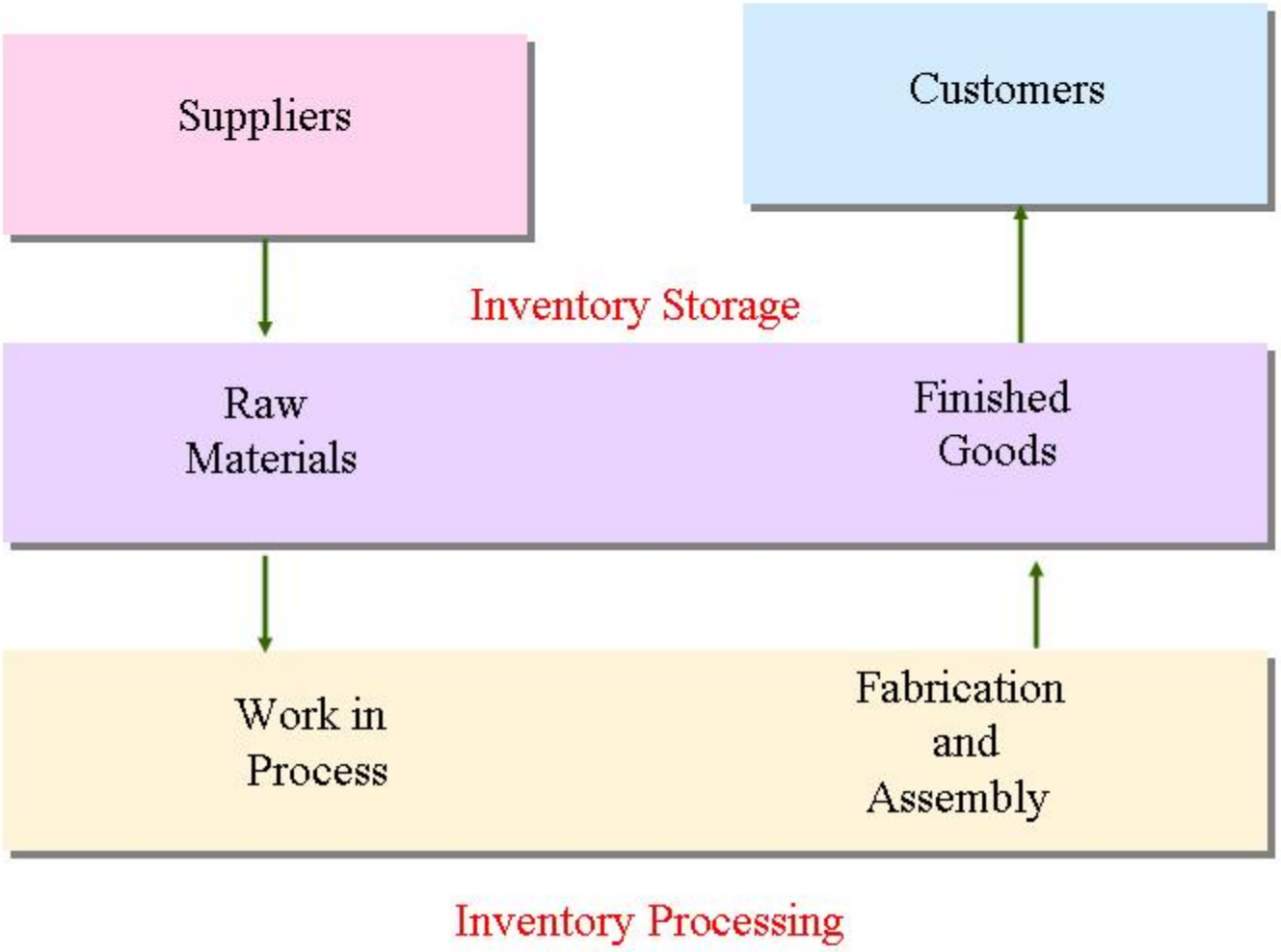
Courtesy , Dr Rafi Javed Qureshi

Inventory Planning and Control



Courtesy , Dr Rafi Javed Qureshi

The Inventory Process

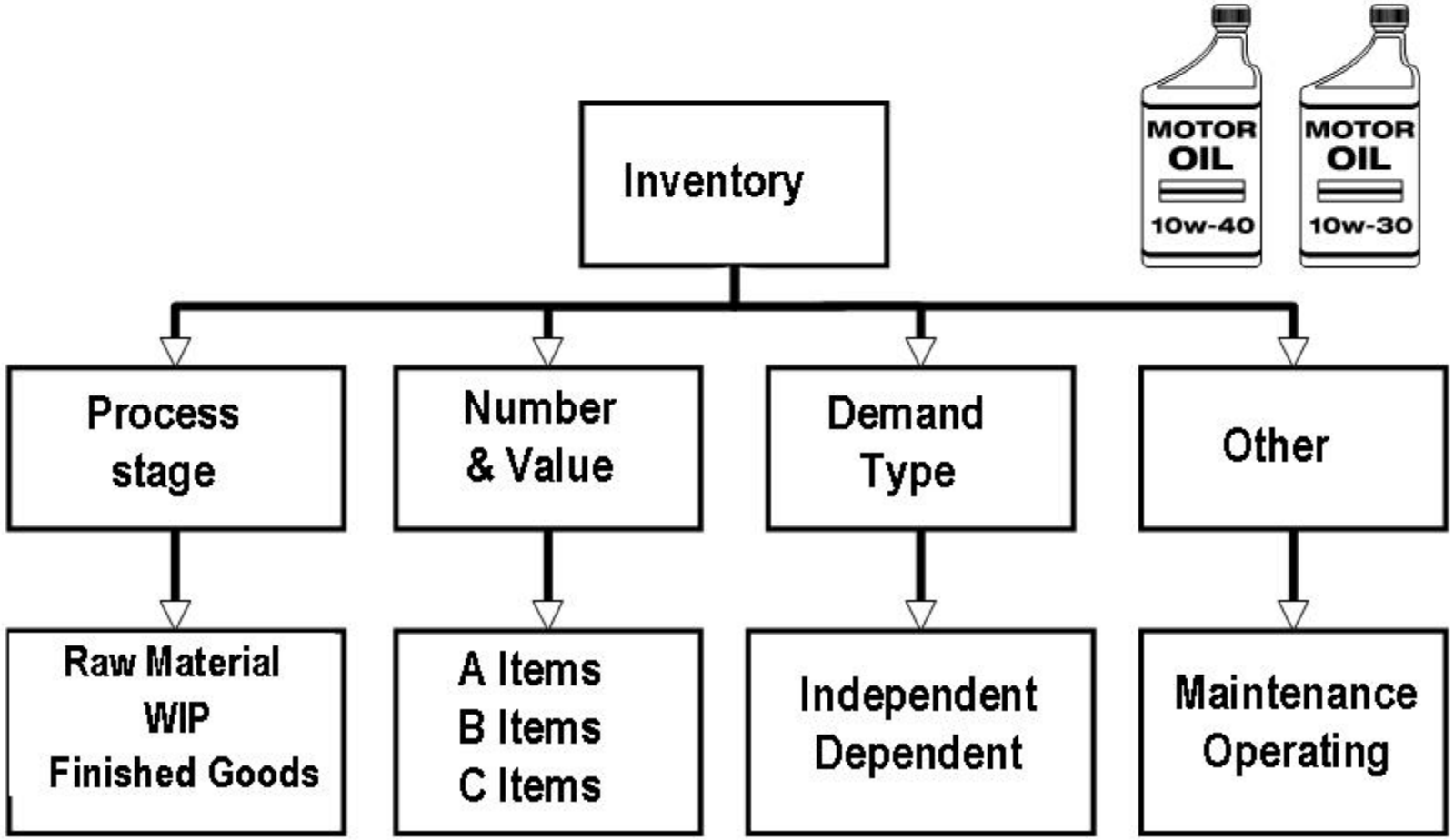


Courtesy , Dr Rafi Javed Qureshi

Types of Inventory

- Raw material
- Work-in-progress
- Maintenance/repair/operating supply
- Finished goods

Inventory Classifications



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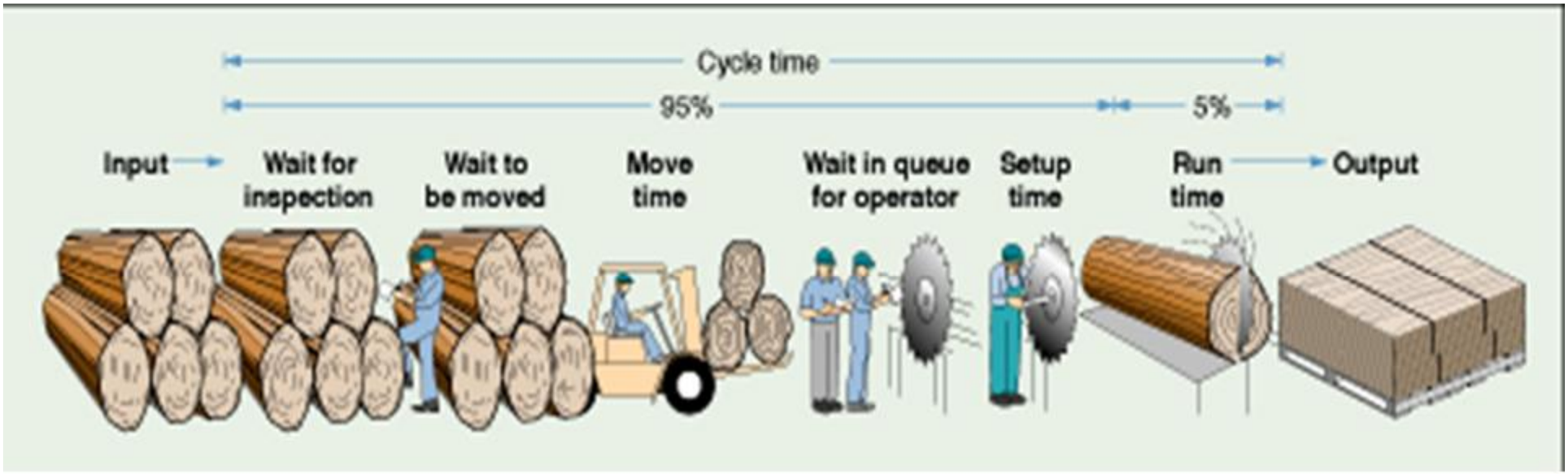
Material Flow Cycle



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ABC Analysis

- ***Class A*** (15% of total inventory, 70-80% dollar usage)
- ***Class B*** (30% of total inventory, 15-25% dollar usage)
- ***Class C*** (55% of total inventory, 5% dollar usage)

Pareto Principle

Courtesy , Dr Rafi Javed Qureshi

Policies based on ABC Analysis

- Resources expended on supplier for **A** items
- **A** items should have tighter control as opposed to **B, C** items
- More care to Forecast of **A** items

10%-25% loss of profit due to inaccurate inventory records

Courtesy , Dr Rafi Javed Qureshi

Inventory Decisions

- How much to order
- When to order

wish to minimize total inventory cost

Inventory Costs

- Cost of the items
- Cost of ordering
- Cost of carrying, or holding inventory
- Cost of safety stock
- Cost of stockouts

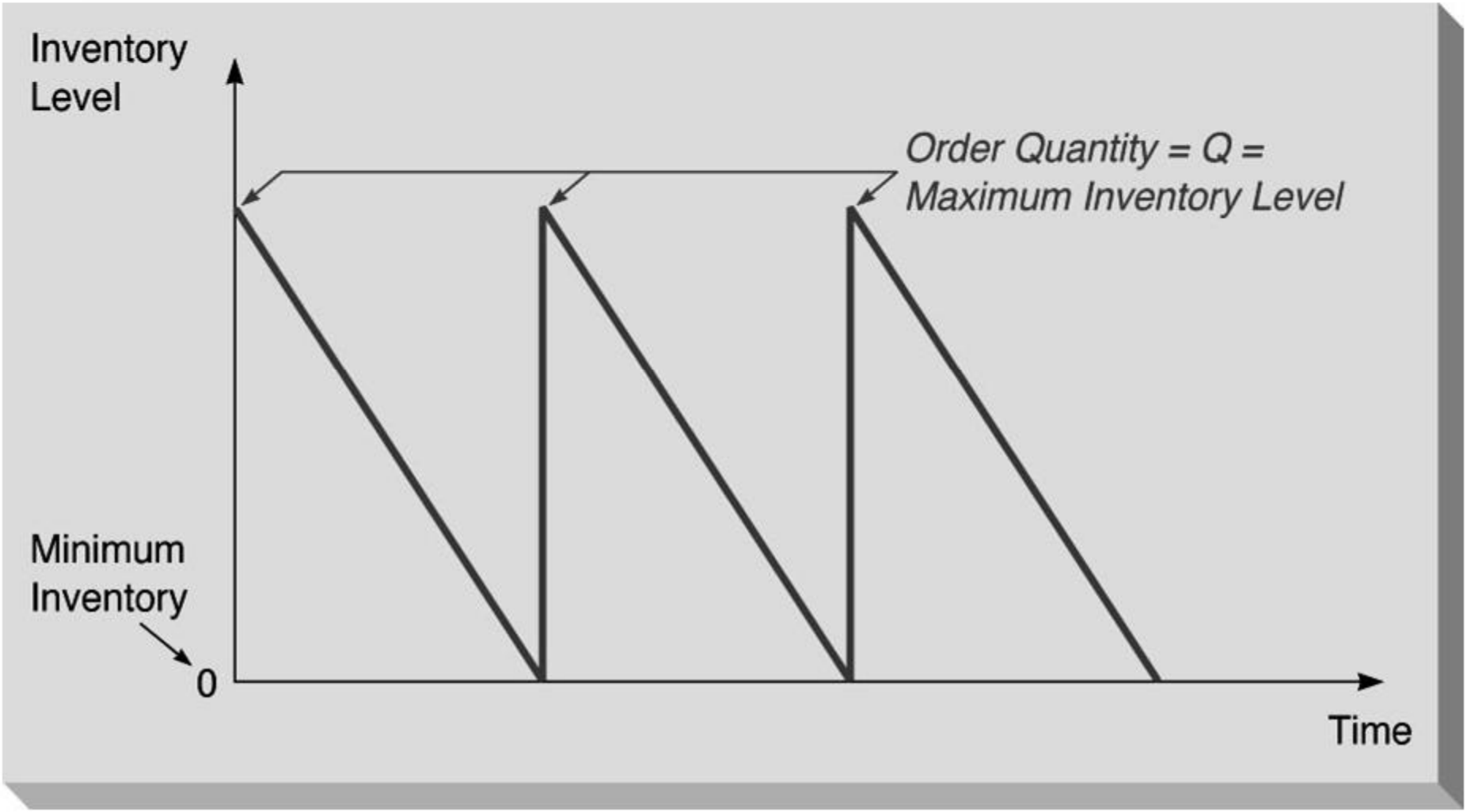
Ordering Costs

- Developing and sending purchase orders
- Processing and inspecting incoming inventory
- Bill paying
- Inventory inquiries
- Utilities, phone bills, etc., - purchasing department.
- Salaries/wages - purchasing department employees
- Supplies (e.g., forms and paper) - purchasing department

Carrying Costs

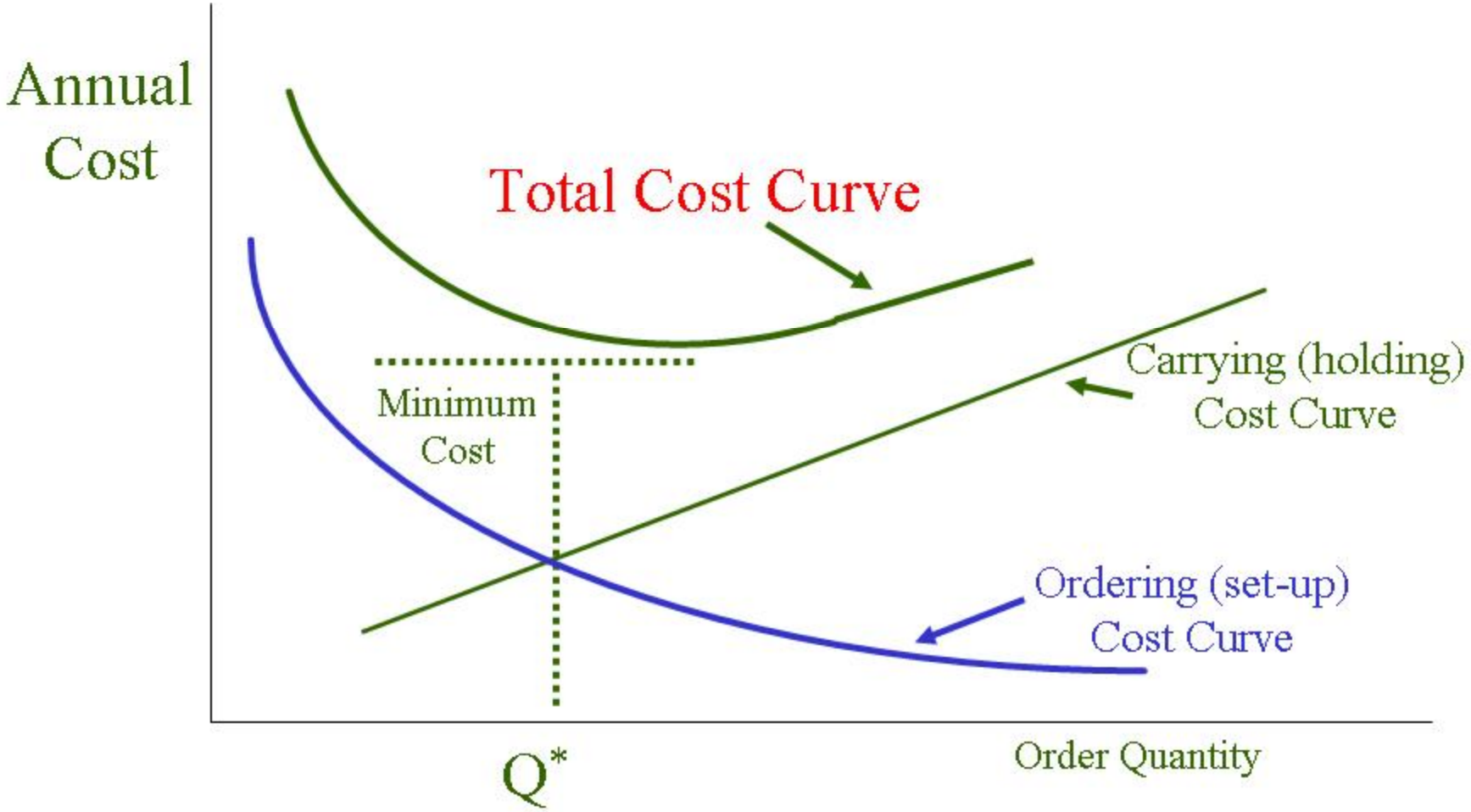
- Cost of capital
- Taxes
- Insurance
- Spoilage
- Theft
- Obsolescence
- Salaries/wages - warehouse employees
- Utilities/building costs - warehouse
- Supplies (e.g., forms, paper) - warehouse

Inventory Usage Over Time



Courtesy , Dr Rafi Javed Qureshi

Costs as Functions of Order Quantity



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Steps in Finding the Optimum Inventory

- Develop an expression for the ordering cost.
- Develop an expression for the carrying cost.
- Set the ordering cost equal to the carrying cost.
- Solve this equation for the optimum desired.

EOQ : Basic Assumptions

- Demand is known and constant
- Lead time is known and constant
- Receipt of inventory is instantaneous
- Quantity discounts are not possible
- The only variable costs are the cost of setting up or placing an order, and the cost of holding or storing inventory over time
- Stockouts can be completely avoided if orders are placed at the appropriate time

Courtesy , Dr Rafi Javed Qureshi

Developing the EOQ

- Annual ordering cost:

$$= \frac{\text{Annual demand}}{\text{Number of units per order}}$$

$$= \frac{D}{Q} C_o$$

- Annual holding or carrying cost:

$$= \text{Average Inventory} * \text{Carrying Cost Per Year}$$

$$= \frac{Q}{2} C_h$$

- Total inventory cost: $C_t = \frac{D}{Q} C_o + \frac{Q}{2} C_h$

Courtesy , Dr Rafi Javed Qureshi

Total cost function is dependent on Q

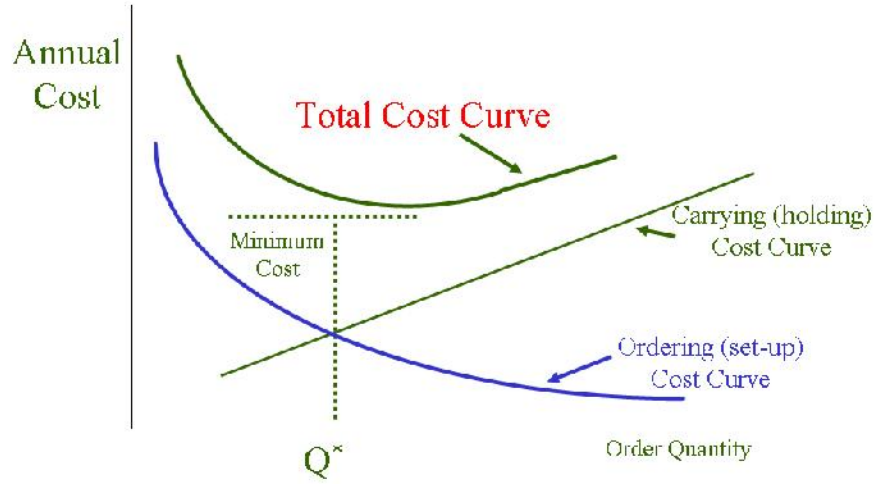
$$C_t(Q) = \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

The value of $C_t(Q)$ is minimum at a point on the cost curve where

$$\frac{\partial C_t}{\partial Q} = 0$$

Hence, partially differentiate $C_t(Q)$ function w.r.t. Q, and, set it equal to zero.

$$\frac{\partial C_t}{\partial Q} = -\frac{DC_o}{Q^2} + \frac{C_h Q}{2} = 0 \rightarrow Q^* = \sqrt{\frac{2DC_o}{C_h}}$$



Q* is called Economic order Quantity

Courtesy , Dr Rafi Javed Qureshi

EOQ

Per Unit Carrying Cost:

$$Q^* = \sqrt{\frac{2DC_0}{C_h}}$$

Percentage Carrying Cost:

$$Q^* = \sqrt{\frac{2DC_0}{IP}}$$

Sometimes, C_h is expressed as;

$$C_h = IP$$

Where *I* = inventory carrying cost rate

***P* = Purchase price of the item**

Courtesy , Dr Rafi Javed Qureshi

Inputs and Outputs of the EOQ Model

Input Values

Annual Demand
(D)

Ordering Cost
(C_o)

Carrying Cost
(C_h)

Lead Time
(L)

Demand Per Day
(d)

EOQ Models

Output Values

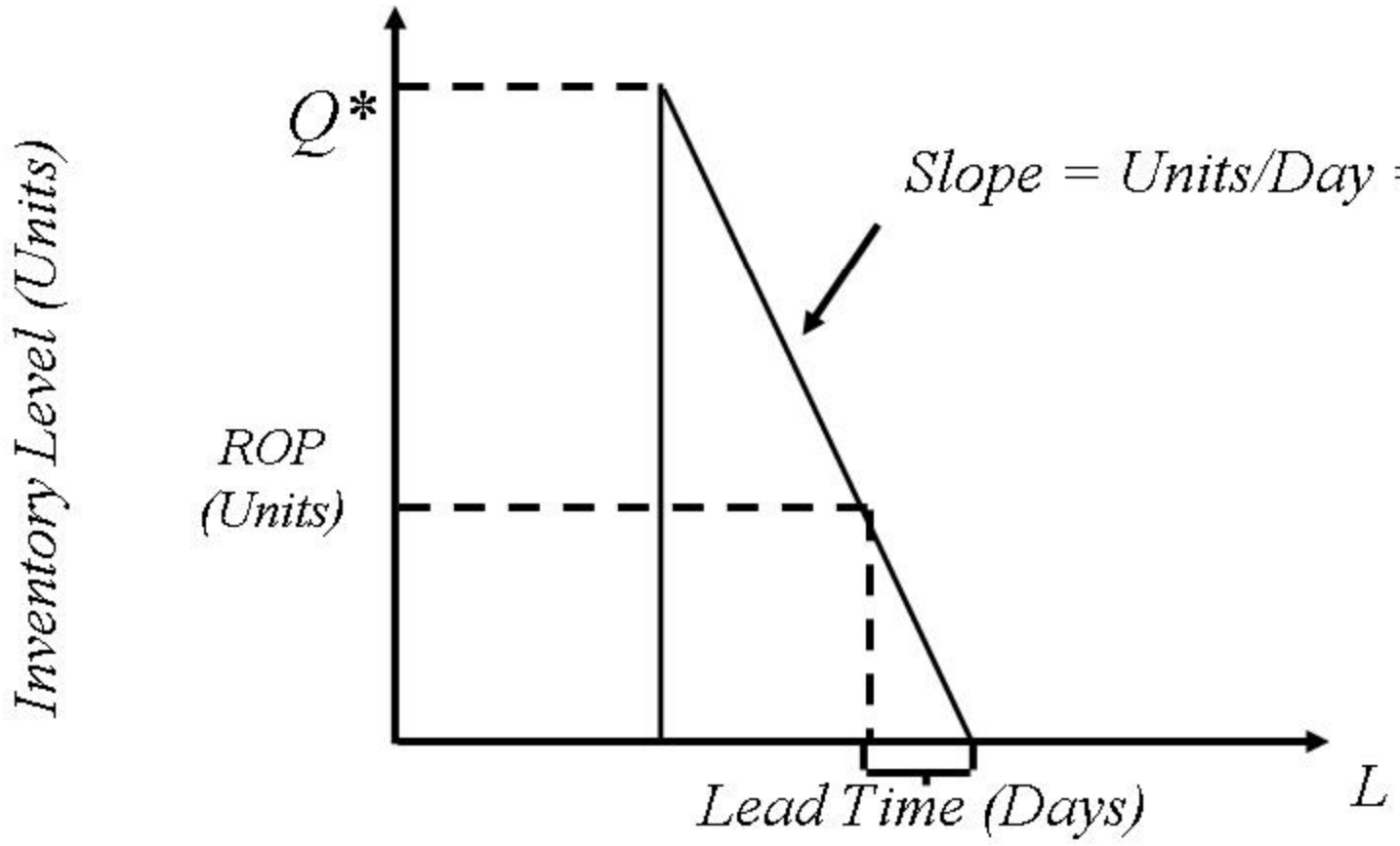
Economic Order Quantity
(EOQ)

Reorder Point
(ROP)

Courtesy , Dr Rafi Javed Qureshi

The Reorder Point (ROP) Curve

$$ROP = (\text{Demand per day}) \times (\text{Lead time for a new order, in days}) = d \times L$$



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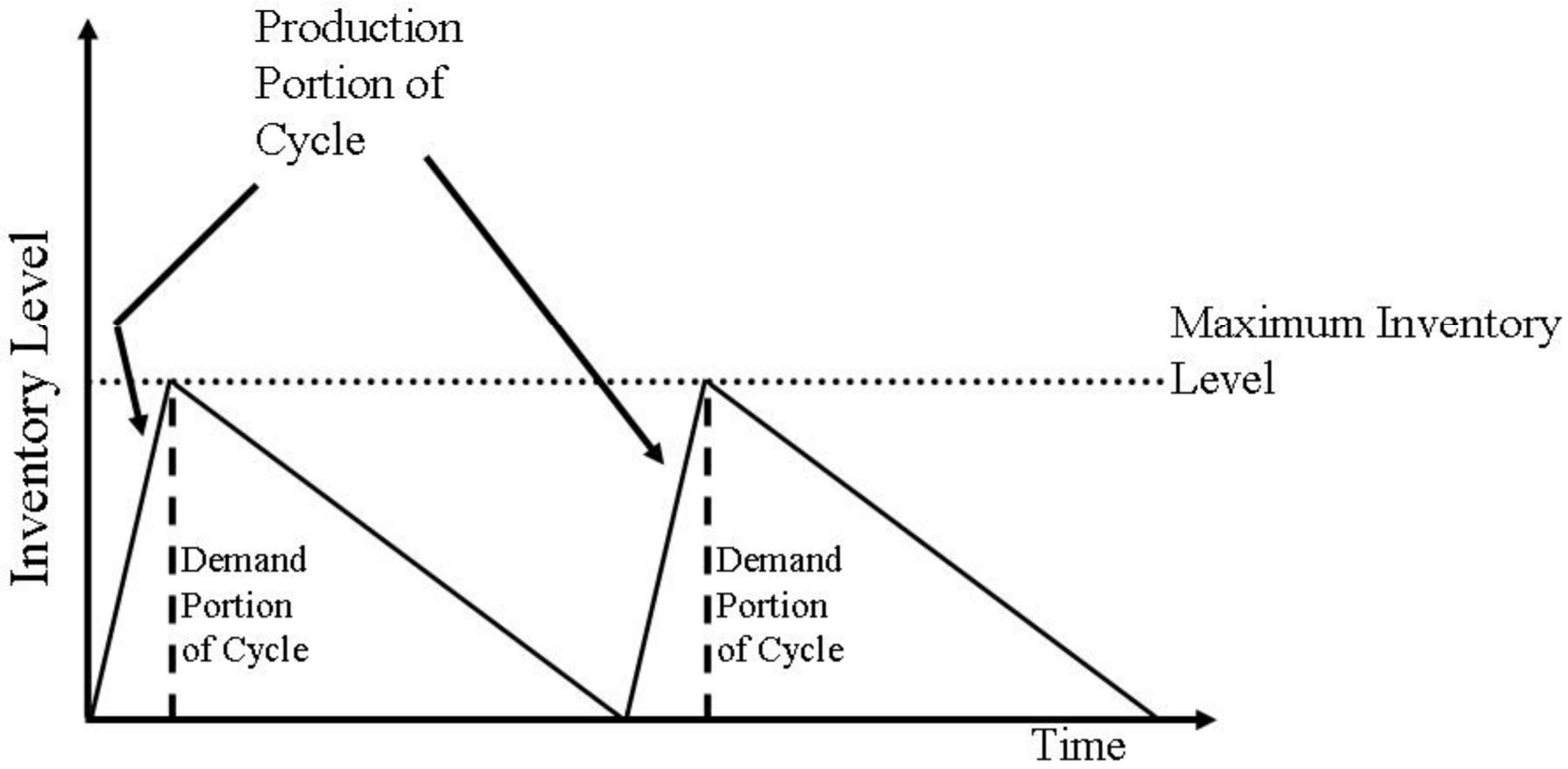
Inventory Control and the Production Process



Courtesy , Dr Rafi Javed Qureshi

Production Quantity EOQ

- Annual Carrying Cost: $\frac{Q}{2}(1-\frac{d}{p})C_h$

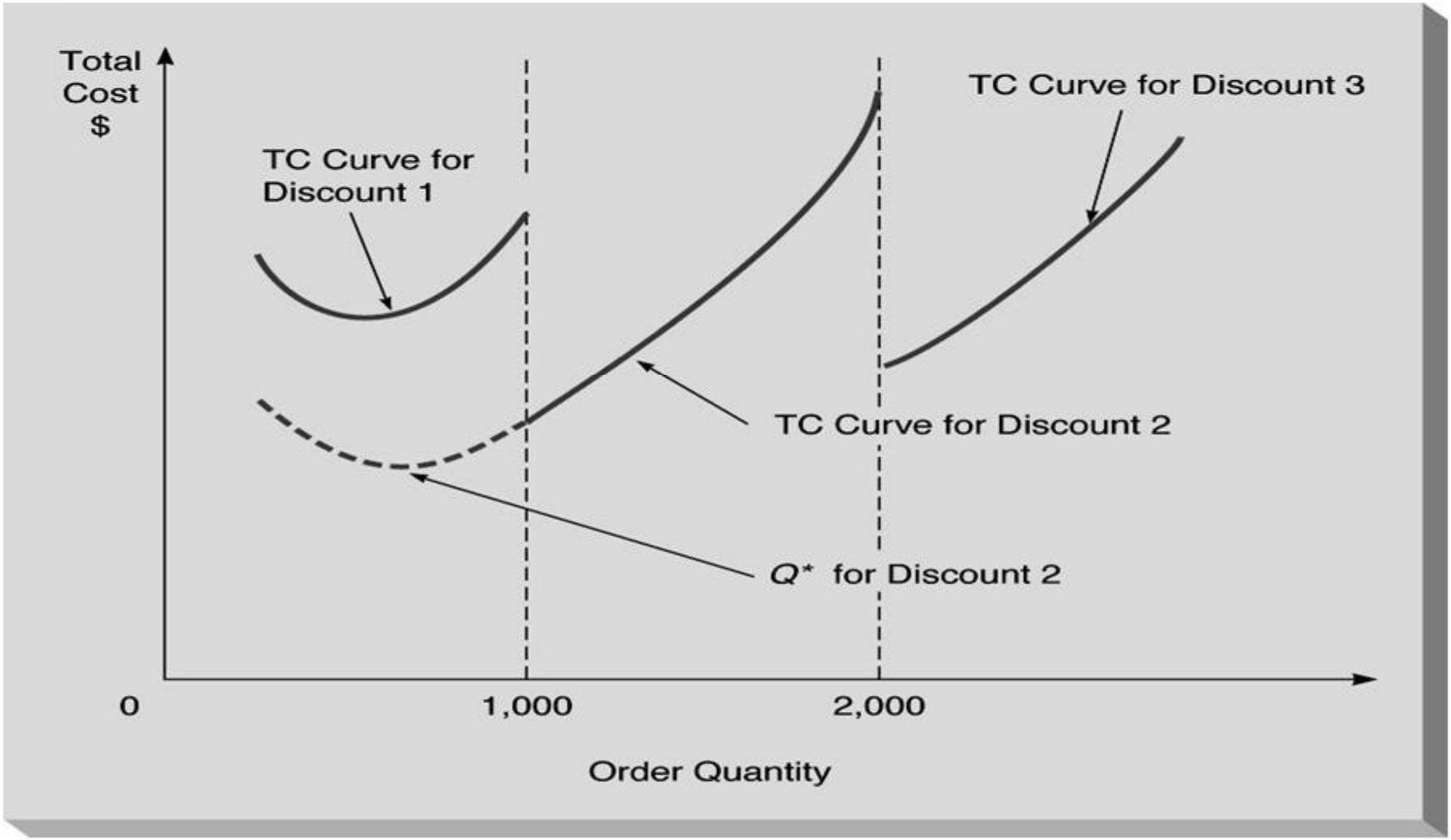
- Annual Ordering Cost:: $\frac{D}{Q}C_s$

Production Quantity EOQ

$$Q_p^* = \sqrt{\frac{2DC_o}{C_h\left(1-\frac{d}{p}\right)}}$$

Courtesy , Dr Rafi Javed Qureshi

Quantity Discount Models



Courtesy , Dr Rafi Javed Qureshi

Quantity Discount Steps

- 1. Calculate Q for each discount
- 2. Adjust Q upward if quantity is too low for discount
- 3. Compute total cost for each discount
- 4. Select Q with the the lowest total cost

Quantity Discounts : Example

ZORIC buys screwed bolts in bulk quantities from 21st century Manufacturing company. This manufacturing company offers quantity discounts to customers who make procurements in large quantities. The discount schedule is as under;

Order Quantity	Price/unit(P)
1 < Q < 1000	$ 5.00
1000 <= Q <2000	4.80
Q >= 2000	4.75

ZORIC's Inventory Manager Zhufaar wants to make decision on order size (Q) so as to minimize his inventory as well as purchase cost of screwed bolts. He estimates that annual demand of the screwed bolts will be 5000. Each order will cost him $49. The inventory carrying cost rate is estimated to be 20%.

In case inventory carrying cost rate turns out to be 10%, what Zhufaar will do?

Quantity Discounts : Example Solution

Let's denote three discount slabs as under;

Discount No	Order Quantity	Price/unit(P)
1	1 < Q < 1000	$ 5.00
2	1000 <= Q <2000	4.80
3	Q >= 2000	4.75

For each discount schedule corresponding EOQ's will be

$$Q_j^* = \sqrt{\frac{2DC_0}{IPj}} \quad \rightarrow \quad Q_j^* = \sqrt{\frac{2(5000)(49)}{(0.20)(P_j)}}$$

Q_1^*= 700 Q_2^* = 714.43 Q_3^*=718.18

Courtesy , Dr Rafi Javed Qureshi

Quantity Discounts : Example Solution

$Q_1^* = 700$ $Q_2^* = 714.43$ $Q_3^* = 718.18$

Fitting the EOQ's in corresponding slab;

Discount No	Order Quantity	EOQ	Adjusted Q
1	1 < Q < 1000	700	700
2	1000 <= Q <2000	714.43	1000
3	Q >= 2000	718.18	2000

Total cost/year = Total Purchase Price of Screwed Bolts + Ordering cost per year + inventory holding cost per year

Mathematically;

$$C_t(Q_j) = P_j D + \frac{DC_o}{Q_j} + \frac{Q_j}{2} IP_j$$

$$C_t(Q_j) = P_j(5000) + \frac{(5000)(49)}{Q_j} + \frac{Q_j(0.20)P_j}{2}$$

Courtesy , Dr Rafi Javed Qureshi

Quantity Discounts & Inventory Carrying Cost Rate

		demand=	5000	inv rate=	0.2	order cost=	49
Discount number	Unit Price	Qj	Order Quantity	Material Cost / yr	Order Cost / yr	Inventory Carrying Cost / yr	TOTAL
1	5.00	700.00	700	25000	350	350	25700
2	4.80	714.43	1000	24000	245	480	24725
3	4.75	718.18	2000	23750	122.5	950	24823

		demand=	5000	inv rate=	0.1	order cost=	49
Discount number	Unit Price	Qj	Order Quantity	Material Cost / yr	Order Cost / yr	Inventory Carrying Cost / yr	TOTAL
1	5.00	989.95	990	25000	247.4747	247.5	25495
2	4.80	1010.36	1010	24000	242.5743	242.4	24485
3	4.75	1015.67	2000	23750	122.5	475	24348

Courtesy: Dr Rafi Javed Qureshi

Quantity Discounts : Example Solution

For I = 20%, least cost order size = 1000 with Discount schedule # 2

For I = 10%, least cost order size = 2000 with Discount schedule # 3

Courtesy , Dr Rafi Javed Qureshi

EOQ APPLICATION

- ARMEDI Business keeps a large stock of items in different parts of the country. The pressure vessel division of the company has a sizable store in their office building. The store keeps 20 different items. Annual demand data of these items along with item purchase cost is tabulated (and presented on next slide). The company follows the following ordering policy;
- “If annual demand of an item > 10,000; make 2 orders per year; otherwise make one order”
- *What is the total inventory cost of 20-item store?*

Courtesy , Dr Rafi Javed Qureshi

20 –Item With Pre-defined Inventory Policy

Replenish inventory once in year if D<=10000, and make two orders otherwise

Inv Carrying cost rate = 20%

Item No	P	Dj	Co	Item Cost	No_of_ orders/yr	Order Size	Ord Cost per year	Inv Carry Cost/yr	Total Cost
1	350	12000	12	4200000	2	6000	24	210000	210024
2	124	3456	6	428544	1	3456	6	42854	42860
3	678	89043	54	60371154	2	44522	108	3018558	3018666
4	321	13489	21	4329969	2	6745	42	216498	216540
5	890	5432	78	4834480	1	5432	78	483448	483526
6	621	56741	43	35236161	2	28371	86	1761808	1761894
7	701	28905	65	20262405	2	14453	130	1013120	1013250
8	243	23412	23	5689116	2	11706	46	284456	284502
9	611	45672	86	27905592	2	22836	172	1395280	1395452
10	711	45290	45	32201190	2	22645	90	1610060	1610150
11	112	567890	12	63603680	2	283945	24	3180184	3180208
12	456	32156	45	14663136	2	16078	90	733157	733247
13	231	8900	20	2055900	1	8900	20	205590	205610
14	932	4320	89	4026240	1	4320	89	402624	402713
15	904	45600	100	41222400	2	22800	200	2061120	2061320
16	145	78000	20	11310000	2	39000	40	565500	565540
17	764	23400	41	17877600	1	23400	41	1787760	1787801
18	903	5432	90	4905096	1	5432	90	490510	490600
19	256	1678	30	429568	1	1678	30	42957	42987
20	834	5432	26	4530288	1	5432	26	453029	453055
				360082519		TOTAL	1432	19958512	19959944
			Million $	360.082519				Million $	20

EOQ APPLICATION

- Arian Muth is manager of the pressure-vessels store. He was horrified to see 30 Million $ figure.
- He approached Judy Brian in the department for help. Judy was Inventory Consultant in the organization. The Consultant advised him to apply EOQ methodology.
- ***How much Arian Muth will be able to save in inventory costs if he applies EOQ methodology?***

Courtesy , Dr Rafi Javed Qureshi

EOQ Approach

Inv Carrying cost rate = 20%

				inv_rate=		0.2			
Item No	Cj	Dj	Co	Item Cost	EOQ	No_of_ orders/yr	Ord Cost per year	Inv Carry Cost/yr	Total Cost
1	350	12000	12	4200000	64.14	187	2244	2244.99	4488.99
2	124	3456	6	428544	40.89	85	510	507.08	1017.08
3	678	89043	54	60371154	266.31	334	18036	18055.59	36091.59
4	321	13489	21	4329969	93.94	144	3024	3015.45	6039.45
5	890	5432	78	4834480	69.00	79	6162	6140.76	12302.76
6	621	56741	43	35236161	198.22	286	12298	12309.16	24607.16
7	701	28905	65	20262405	163.71	177	11505	11476.31	22981.31
8	243	23412	23	5689116	148.86	157	3611	3617.31	7228.31
9	611	45672	86	27905592	253.54	180	15480	15491.55	30971.55
10	711	45290	45	32201190	169.31	268	12060	12037.66	24097.66
11	112	567890	12	63603680	780.03	728	8736	8736.38	17472.38
12	456	32156	45	14663136	178.14	181	8145	8123.06	16268.06
13	231	8900	20	2055900	87.78	101	2020	2027.76	4047.76
14	932	4320	89	4026240	64.23	67	5963	5986.11	11949.11
15	904	45600	100	41222400	224.59	203	20300	20303.30	40603.30
16	145	78000	20	11310000	328.00	238	4760	4756.05	9516.05
17	764	23400	41	17877600	112.06	209	8569	8561.43	17130.43
18	903	5432	90	4905096	73.58	74	6660	6644.24	13304.24
19	256	1678	30	429568	44.34	38	1140	1135.21	2275.21
20	834	5432	26	4530288	41.15	132	3432	3432.02	6864.02
				360082519			154655		309256.43
			Million $	360.082519				Million $	0.31

Total Inventory Carrying Costs

Total Inventory Costs reduction from

$ 20 Million

To

$0.31 Millions

Courtesy , Dr Rafi Javed Qureshi

Store Space Required under current ordering policy

- Arian Muth was hesitant to apply EOQ-based strategy; as the number of orders per year was going to increase at a drastic rate.
- Arian expressed his reservations about EOQ solutions to Judy. Judy asked him to calculate the present space requirements for 20-items
- Arian collected information about space requirements (in CFT) for all 20-items (shown in next slide).

- ***Based on his present ordering policy, how much space is being utilized by these 20 items in the stores.***

Courtesy , Dr Rafi Javed Qureshi

STORAGE VOLUME *For 20 –Item With Pre-defined Inventory Policy*

Replenish inventory once in year if D<=10000, and make two orders otherwise

Inv Carrying cost rate = 20%

Item No	Cj	Dj	Co	Item Cost	No_of_ orders/yr	Order Size	Warehouse Space /item (CFT)	Total CFT
1	350	12000	12	4200000	2	6000	12	72000
2	124	3456	6	428544	1	3456	24	82944
3	678	89043	54	60371154	2	44522	20	890430
4	321	13489	21	4329969	2	6745	16	107912
5	890	5432	78	4834480	1	5432	8	43456
6	621	56741	43	35236161	2	28371	12	340446
7	701	28905	65	20262405	2	14453	34	491385
8	243	23412	23	5689116	2	11706	14	163884
9	611	45672	86	27905592	2	22836	45	1027620
10	711	45290	45	32201190	2	22645	16	362320
11	112	567890	12	63603680	2	283945	34	9654130
12	456	32156	45	14663136	2	16078	45	723510
13	231	8900	20	2055900	1	8900	32	284800
14	932	4320	89	4026240	1	4320	64	276480
15	904	45600	100	41222400	2	22800	20	456000
16	145	78000	20	11310000	2	39000	24	936000
17	764	23400	41	17877600	1	23400	36	842400
18	903	5432	90	4905096	1	5432	30	162960
19	256	1678	30	429568	1	1678	32	53696
20	834	5432	26	4530288	1	5432	12	65184
				360082519		TOTAL	Warehouse Space (CFT)	17037557

Courtesy , Dr Rafi Javed Qureshi

Store Space Required under EOQ policy

- Arian Muth got terrified to see this horrendous figure of 17037557 cubic feet (CFT) being currently utilized for storing those 20-items.
- Arian spoke to Judy again telling this huge amount of space required to store these 20-items
- Judy advised him to make storage space requirement calculations using EOQ approach.

- ***Based on EOQ policy, how much space is being saved?***

Courtesy , Dr Rafi Javed Qureshi

STORAGE VOLUME For EOQ Approach

Inv Carrying cost rate = 20%

Item No	Cj	Dj	Co	Item Cost	EOQ	Warehouse Space /item (CFT)	Total CFT
1	350	12000	12	4200000	64.14	12	770
2	124	3456	6	428544	40.89	24	981
3	678	89043	54	60371154	266.31	20	5326
4	321	13489	21	4329969	93.94	16	1503
5	890	5432	78	4834480	69.00	8	552
6	621	56741	43	35236161	198.22	12	2379
7	701	28905	65	20262405	163.71	34	5566
8	243	23412	23	5689116	148.86	14	2084
9	611	45672	86	27905592	253.54	45	11409
10	711	45290	45	32201190	169.31	16	2709
11	112	567890	12	63603680	780.03	34	26521
12	456	32156	45	14663136	178.14	45	8016
13	231	8900	20	2055900	87.78	32	2809
14	932	4320	89	4026240	64.23	64	4111
15	904	45600	100	41222400	224.59	20	4492
16	145	78000	20	11310000	328.00	24	7872
17	764	23400	41	17877600	112.06	36	4034
18	903	5432	90	4905096	73.58	30	2207
19	256	1678	30	429568	44.34	32	1419
20	834	5432	26	4530288	41.15	12	494
				360082519		Warehouse Space (CFT)	95255

Courtesy , Dr Rafi Javed Qureshi

STORAGE VOLUME REDUCTION

Total Reduction from

17035757 Cubic Feet

To

95255 Cubic Feet

Courtesy , Dr Rafi Javed Qureshi

How much Dollars required for making purchases

- Inventory consultant Judy wanted to convince store manager Arian on another very beneficial aspect of EOQ policy.
- Judy asked Arian Muth to compare the dollars required to invest at the time of placing orders for both policies.
- Dollars invested at the time of placing order is equal to

$$\sum_{j=1}^{20} P_j Q_j$$

where, P_j = purchase price of j^{th} item

Q_j = order size of j^{th} item

- ***Compare the Dollars requirement for both policies?***

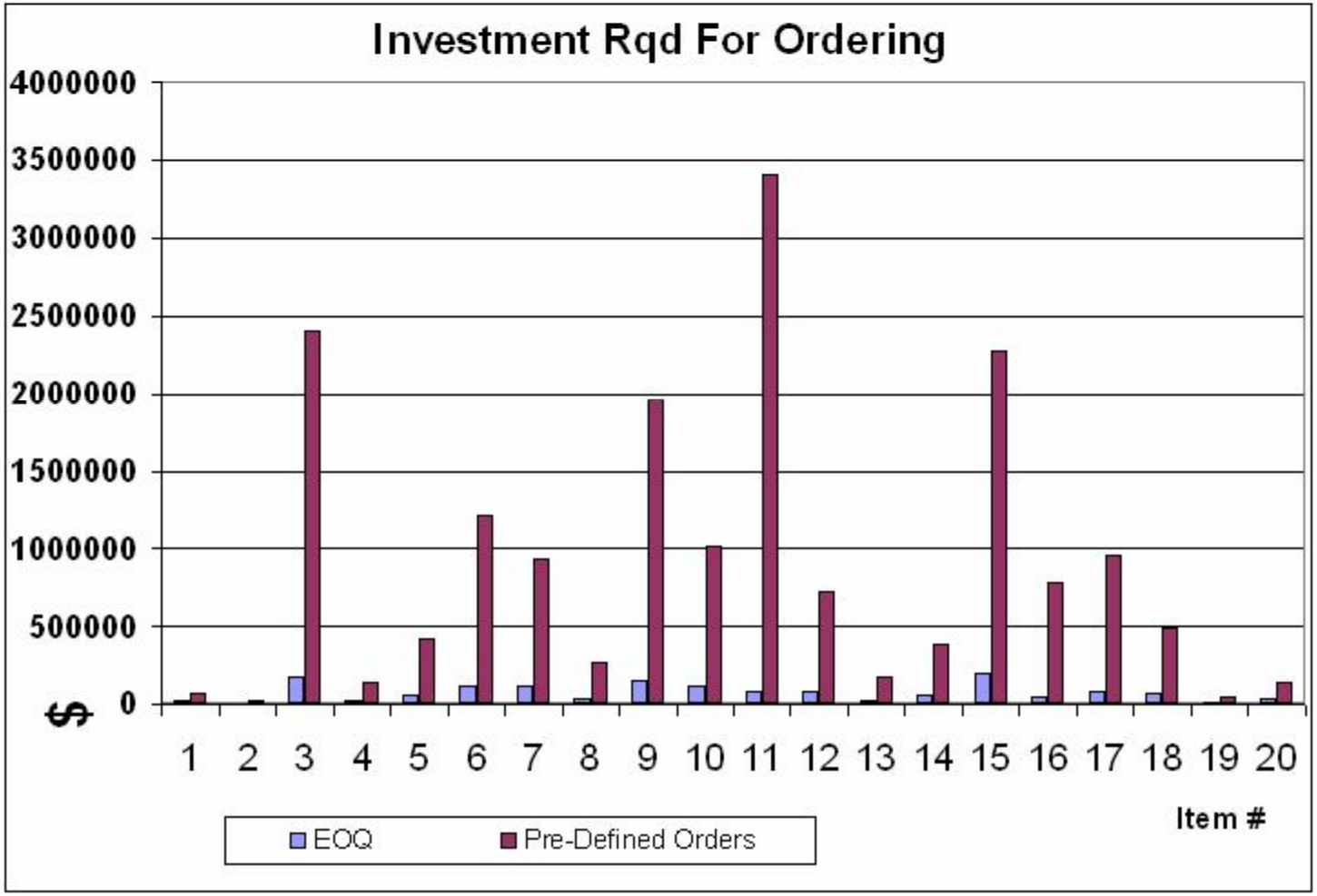
Courtesy , Dr Rafi Javed Qureshi

$ Required at the time of Making an order

Item No	Cj	Dj	Co	Item Cost	EOQ	Investment Dollars Rqd	Order Size	Investment Dollars Rqd
1	350	12000	12	4200000	64.14	22450	6000	72000
2	124	3456	6	428544	40.89	5071	3456	20736
3	678	89043	54	60371154	266.31	180556	44522	2404161
4	321	13489	21	4329969	93.94	30154	6745	141635
5	890	5432	78	4834480	69.00	61408	5432	423696
6	621	56741	43	35236161	198.22	123092	28371	1219932
7	701	28905	65	20262405	163.71	114763	14453	939413
8	243	23412	23	5689116	148.86	36173	11706	269238
9	611	45672	86	27905592	253.54	154915	22836	1963896
10	711	45290	45	32201190	169.31	120377	22645	1019025
11	112	567890	12	63603680	780.03	87364	283945	3407340
12	456	32156	45	14663136	178.14	81231	16078	723510
13	231	8900	20	2055900	87.78	20278	8900	178000
14	932	4320	89	4026240	64.23	59861	4320	384480
15	904	45600	100	41222400	224.59	203033	22800	2280000
16	145	78000	20	11310000	328.00	47560	39000	780000
17	764	23400	41	17877600	112.06	85614	23400	959400
18	903	5432	90	4905096	73.58	66442	5432	488880
19	256	1678	30	429568	44.34	11352	1678	50340
20	834	5432	26	4530288	41.15	34320	5432	141232

Courtesy , Dr Rafi Javed Qureshi

$ Required At Ordering Time



Courtesy , Dr Rafi Javed Qureshi

EOQ Challenges

- Arian Muth was finally convinced that EOQ is highly valuable tool for inventory control.
- *His main concern was high values of frequent ordering activity as a result of EOQ implementation.*
- Judy advised him to make his case before Director of Pressure-Vessels cell. In cooperation with Director Purchases, Arian was able to activate Purchase department to adapt to this frequent ordering requirements to implement the new inventory control strategy.

Courtesy , Dr Rafi Javed Qureshi